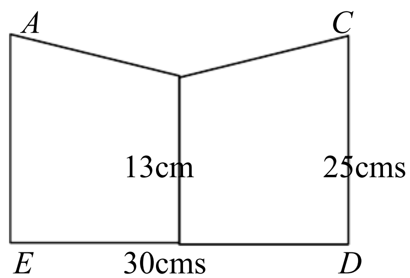
MECHANICS (C) UNIT 2 TEST PAPER 5

Take $g = 9{\cdot}8\ \text{ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A pump raises water from a reservoir at a depth of 25 m below ground level. The water is delivered at ground level with speed 12 ms^{-1} through a pipe of radius 4 cm. Find
   (i) the potential and kinetic energy given to the water each second, [4]
   (ii) the rate, in kW, at which the pump is working. [2]
   [1 m^3 of water has a mass of 1000 kg.]

2. Aliya, whose mass is m kg, is playing rounders. She rounds the first base at a speed of v ms^{-1}, making the turn on a horizontal circular path of radius r m.
   (i) Write down, in terms of m, v and r, the magnitude of the horizontal force acting on her. [1]
   (ii) Show that if she continues on the same circular path, the reaction force exerted on her by the ground must act at an angle θ to the vertical, where $\tan\theta = \dfrac{v^2}{gr}$. [6]

3. 

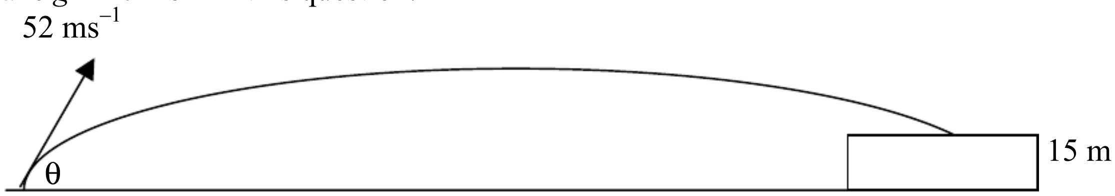
The diagram shows a uniform lamina *ABCDE* formed by removing a symmetrical triangular section from a rectangular sheet of metal measuring 30 cm by 25 cm.
   (i) Find the distance of the centre of mass of the lamina from *ED*. [3]

   The lamina has mass m. A particle P is attached to the lamina at B.
   The lamina is then suspended freely from A and hangs in equilibrium with AD vertical.
   (ii) Find, in terms of m, the mass of P. [5]

4. A car, of mass 1100 kg, pulls a trailer of mass 550 kg along a straight horizontal road by means of a rigid tow-bar. The car is accelerating at $1{\cdot}2\ \text{ms}^{-2}$ and the resistances to the motion of the car and trailer have magnitudes 500 N and 200 N respectively.
   (i) Show that the driving force produced by the engine of the car is 2680 N. [3]
   (ii) Find the tension in the tow-bar between the car and the trailer. [2]
   (iii) Find the rate, in kW, at which the car's engine is working when the car is moving with speed 18 ms^{-1}. [2]

   When the car is moving at 18 ms^{-1} it starts to climb a straight hill which is inclined at 6^0 to the horizontal. If the car's engine continues to work at the same rate and the resistances to motion remain the same as previously,
   (iv) find the acceleration of the car at the instant when it starts to climb the hill. [3]
   (v) Show that tension in the tow-bar remains unchanged. [2]

5. Take $g = 10$ ms^{-2} in this question.



A golfer hits a ball from a point T at an angle θ to the horizontal, where $\sin\theta = {}^{5}/_{13}$, giving it an initial speed of 52 ms^{-1}. The ball lands on top of a mound, 15 m above the level of T, as shown.

(i) Show that the height, y m, of the ball above T at time t seconds after it was hit is given by

$$y = 20t - 5t^2.$$ [3]

(ii) Find the time for which the ball is in flight. [3]

(iii) Find the horizontal distance travelled by the ball. [2]

(iv) Show that, if the ball is x m horizontally from T at time t seconds, then

$$y = \frac{5}{12}x - \frac{5}{2304}x^2.$$ [3]

(v) Name a force that has been ignored in your mathematical model and state whether the answer to part (ii) would be larger or smaller if this force were taken into account. [2]

6. Two smooth spheres, A and B, of equal radius but of masses $3m$ and $4m$ respectively, are free to move in a straight horizontal groove. The coefficient of restitution between them is e.
A is projected with speed u to hit B, which is initially at rest.

(i) Show that B begins to move with speed ${}^{3}/_{7}\, u(1 + e)$. [5]

(ii) Given that A is brought to rest by the collision, show that $e = 0{\cdot}75$. [3]

Having been brought to rest, A is now set in motion again by being given an impulse of magnitude kmu Ns, where $k > 2{\cdot}25$. A then collides again with B.

(iii) Show that the speed of A after this second impact is independent of k. [6]

MECHANICS 2 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1.	(i) Volume per second = $12\pi(0{\cdot}04)^2 = 0{\cdot}0603$ m^3 Mass= 60·3 kg	M1 A1
	P.E. gained per sec. = $60{\cdot}3 \times g \times 25 = 14\,778$ J	A1
	K.E. gained per sec. = $\frac{1}{2} \times 60{\cdot}3 \times 12^2 = 4342$ J	A1
	(ii) Power = total energy per second = 19 120 Js^{-1} = 19·1 kW	M1 A1 6
2.	(i) Resultant force towards centre = mv^2/r	B1
	(ii) Forces acting on her are vertical weight = mg, reaction R at θ to vertical.	B1

		Contact with ground maintained, so $R\cos\theta = mg$	B1 B1	
		Horizontally : $R\sin\theta = mv^2/r$ Divide : $\tan\theta = v^2/gr$	B1 M1 A1	7
3.	(i)	$750(12{\cdot}5) = 180(21) + 570\bar{y}$ $\bar{y} = 9{\cdot}82$ cm	M1 A1 A1	
	(ii)	Must have centre of mass 12·5 cm from ED	B1	
		$9{\cdot}816m + 13M = 12{\cdot}5(m + M)$ $0{\cdot}5M = 2{\cdot}684m$ $M = 5{\cdot}37m$	M1 A1 M1 A1	8
4.	(i)	$F - 700 = 1650 \times 1{\cdot}2$ $F = 700 + 1980 = 2680$ N	M1 A1 A1	
	(ii)	$F - 500 - T = 1100 \times 1{\cdot}2$ $T = 2180 - 1320 = 860$ N	M1 A1	
	(iii)	$P = 2680 \times 18 = 48{\cdot}2$ kW	M1 A1	
	(iv)	$48240 = 18(700 + 1650g\sin 6^0 + 1650a)$ $a = 0{\cdot}176$ ms^{-2}	M1 A1 A1	
	(v)	For trailer, $T - 200 - 550g\sin 6^0 = 550(0{\cdot}176)$ $T = 860$ N	M1 A1	12
5.	(i)	$y = (52\sin\theta)t - \frac{1}{2}gt^2 = 20t - 5t^2$	M1 A1 A1	
	(ii)	Lands when $y = 15$ $t^2 - 4t + 3 = 0$ $(t-1)(t-3) = 0$	M1 A1	
		Ball is coming down, so $t = 3$	A1	
	(iii)	$x = (52\cos\theta)t = 52 \times {}^{12}/_{13}\, t = 48t$ When $t = 3$, $x = 144$ m	M1 A1	
	(iv)	$y = 20\times\frac{x}{48} - 5\times\left(\frac{x}{48}\right)^2 = \frac{5}{12}x - \frac{5}{2304}x^2$	M1 M1 A1	
	(v)	Have ignored air resistance, which would make answer larger	B1 B1	13
6.	(i)	Momentum : $3mu = 3mv_A + 4mv_B$ $3v_A + 4v_B = 3u$	B1	
		Elasticity : $(v_B - v_A)/(-u) = -e$ $3v_B - 3v_A = 3eu$	M1 A1	
		Add : $3u(1 + e) = 7v_B$ $v_B = {}^3/_7 u(1 + e)$	M1 A1	
	(ii)	If $v_A = 0$, $v_B = eu$ and $4v_B = 3u$, so $e = 0{\cdot}75$	M1 A1 A1	
	(iii)	Now A has speed $\frac{1}{3}ku$ $(v'_B - v'_A)/(0{\cdot}75u - \frac{1}{3}ku) = -0{\cdot}75$	M1 A1	
		and $kmu + 3mu = 3mv'_A + 4mv'_B$	B1	
		$ku + 3u = 3v'_A + 4(v'_A - 0{\cdot}75(0{\cdot}75 - \frac{1}{3}k)u) = 7v'_A - 2{\cdot}25u + ku$	M1 A1	
		$v'_A = 0{\cdot}75u$, which is independent of k	A1	14